Leis do Cálculo Funcional (2019/20)

FUNÇÕES

Natural-id	$f \cdot id = id \cdot f = f$	(1)
Assoc-comp	$(f \cdot g) \cdot h \quad = \quad f \cdot (g \cdot h)$	(2)
Natural-const	$\underline{k} \cdot f \quad = \quad \underline{k}$	(3)
Fusão-const	$f \cdot \underline{k} \quad = \quad \underline{f\ k}$	(4)
Leibniz	$f \cdot h \; = \; g \cdot h \quad \Leftarrow \quad f = g$	(5)

PRODUTO

Universal-$\times$	$k = \langle f, g \rangle \quad \Leftrightarrow \quad \left\{ \begin{array}{l} \pi_1 \cdot k = f \\ \pi_2 \cdot k = g \end{array} \right.$	(6)
Cancelamento-$\times$	$\left\{ \begin{array}{l} \pi_1 \cdot \langle f, g \rangle = f \\ \pi_2 \cdot \langle f, g \rangle = g \end{array} \right.$	(7)
Reflexão-$\times$	$\langle \pi_1, \pi_2 \rangle \quad = \quad id_{A \times B}$	(8)
Fusão-$\times$	$\langle g, h \rangle \cdot f = \langle g \cdot f, h \cdot f \rangle$	(9)
Def-$\times$	$f \times g = \langle f \cdot \pi_1, g \cdot \pi_2 \rangle$	(10)
Absorção-$\times$	$(i \times j) \cdot \langle g, h \rangle = \langle i \cdot g, j \cdot h \rangle$	(11)
Natural-π_1	$\pi_1 \cdot (f \times g) \quad = \quad f \cdot \pi_1$	(12)
Natural-π_2	$\pi_2 \cdot (f \times g) \quad = \quad g \cdot \pi_2$	(13)
Functor-$\times$	$(g \cdot h) \times (i \cdot j) \quad = \quad (g \times i) \cdot (h \times j)$	(14)
Functor-id-$\times$	$id_A \times id_B \quad = \quad id_{A \times B}$	(15)
Eq-$\times$	$\langle f, g \rangle = \langle h, k \rangle \;\Leftrightarrow\; \left\{ \begin{array}{l} f = h \\ g = k \end{array} \right.$	(16)

COPRODUTO

Universal-$+$	$k = [f\ , g] \quad \Leftrightarrow \quad \left\{ \begin{array}{l} k \cdot i_1 = f \\ k \cdot i_2 = g \end{array} \right.$	(17)
Cancelamento-$+$	$\left\{ \begin{array}{l} [f\ , g] \cdot i_1 = f \\ [f\ , g] \cdot i_2 = g \end{array} \right.$	(18)
Reflexão-$+$	$[i_1\ , i_2] \quad = \quad id_{A+B}$	(19)
Fusão-$+$	$f \cdot [g\ , h] = [f \cdot g\ , f \cdot h]$	(20)
Def-$+$	$f + g = [i_1 \cdot f\ , i_2 \cdot g]$	(21)
Absorção-$+$	$[g\ , h] \cdot (i + j) = [g \cdot i\ , h \cdot j]$	(22)
Natural-i_1	$(i + j) \cdot i_1 \quad = \quad i_1 \cdot i$	(23)
Natural-i_2	$(i + j) \cdot i_2 \quad = \quad i_2 \cdot j$	(24)
Functor-$+$	$(g \cdot h) + (i \cdot j) = (g + i) \cdot (h + j)$	(25)
Functor-id-$+$	$id_A + id_B = id_{A+B}$	(26)
Eq-$+$	$[f\ , g] = [h\ , k] \;\Leftrightarrow\; \left\{ \begin{array}{l} f = h \\ g = k \end{array} \right.$	(27)

MISC. PRODUTO / COPRODUTO

Lei da troca	$[\langle f,g\rangle\ ,\langle h,k\rangle] \quad = \quad \langle [f\ ,h],[g\ ,k]\rangle$	(28)

CONDICIONAL

Natural-guarda	$p? \cdot f \quad = \quad (f+f)\cdot(p\cdot f)?$	(29)
Def condicional de McCarthy	$p \rightarrow f,g \quad = \quad [f\ ,g]\cdot p?$	(30)
1.ª Lei de fusão do condicional	$f\cdot(p\rightarrow g,h) \quad = \quad p\rightarrow f\cdot g, f\cdot h$	(31)
2.ª Lei de fusão do condicional	$(p\rightarrow f,g)\cdot h \quad = \quad (p\cdot h)\rightarrow (f\cdot h),(g\cdot h)$	(32)

EXPONENCIAÇÃO

Universal-exp	$k = \overline{f} \quad \Leftrightarrow \quad f = ap\cdot(k\times id)$	(33)
Cancelamento-exp	$f = ap\cdot(\overline{f}\times id)$	(34)
Reflexão-exp	$\overline{ap} = id_{B^A}$	(35)
Fusão-exp	$\overline{g\cdot(f\times id)} = \overline{g}\cdot f$	(36)
Def-exp	$f^A = \overline{f\cdot ap}$	(37)
Absorção-exp	$f^A\cdot\overline{g} = \overline{f\cdot g}$	(38)
Functor-exp	$(g\cdot h)^A = g^A\cdot h^A$	(39)
Functor-id-exp	$id^A = id$	(40)

FUNCTORES

Functor-F	$\mathsf{F}(g\cdot h) \quad = \quad (\mathsf{F}\,g)\cdot(\mathsf{F}\,h)$	(41)
Functor-id-F	$\mathsf{F}\,id_A \quad = \quad id_{(\mathsf{F}\,A)}$	(42)

INDUÇÃO

Universal-cata	$k = (\!\mid g \mid\!) \quad \Leftrightarrow \quad k\cdot\mathsf{in} = g\cdot\mathsf{F}\,k$	(43)
Cancelamento-cata	$(\!\mid g \mid\!)\cdot\mathsf{in} = g\cdot\mathsf{F}\,(\!\mid g \mid\!)$	(44)
Reflexão-cata	$(\!\mid \mathsf{in} \mid\!) = id_{\mathsf{T}}$	(45)
Fusão-cata	$f\cdot(\!\mid g \mid\!) = (\!\mid h \mid\!) \quad \Leftarrow \quad f\cdot g = h\cdot\mathsf{F}\,f$	(46)
Base-cata	$\mathsf{F}\,f \quad = \quad \mathsf{B}\,(id,f)$	(47)
Def-map-cata	$\mathsf{T}\,f \quad = \quad (\!\mid \mathsf{in}\cdot\mathsf{B}(f,id) \mid\!)$	(48)
Absorção-cata	$(\!\mid g \mid\!)\cdot\mathsf{T}\,f \quad = \quad (\!\mid g\cdot\mathsf{B}(f,id) \mid\!)$	(49)

RECURSIVIDADE MÚTUA

Fokkinga	$\left\{\begin{array}{l} f\cdot in = h\cdot\mathsf{F}\,\langle f,g\rangle \\ g\cdot in = k\cdot\mathsf{F}\,\langle f,g\rangle \end{array}\right. \quad \equiv \quad \langle f,g\rangle = (\!\mid \langle h,k\rangle \mid\!)$	(50)
“Banana-split”	$\langle (\!\mid i \mid\!), (\!\mid j \mid\!)\rangle \quad = \quad (\!\mid (i\times j)\cdot\langle \mathsf{F}\,\pi_1, \mathsf{F}\,\pi_2\rangle \mid\!)$	(51)

COINDUÇÃO

Universal-ana	$k = [\![g]\!] \quad \Leftrightarrow \quad \mathsf{out} \cdot k = (\mathsf{F}\, k) \cdot g$	(52)
Cancelamento-ana	$\mathsf{out} \cdot [\![g]\!] = \mathsf{F}\, [\![g]\!] \cdot g$	(53)
Reflexão-ana	$[\![\mathsf{out}]\!] = id_{\mathsf{T}}$	(54)
Fusão-ana	$[\![g]\!] \cdot f = [\![h]\!] \quad \Leftarrow \quad g \cdot f = (\mathsf{F}\, f) \cdot h$	(55)
Base-ana	$\mathsf{F}\, f \quad = \quad \mathsf{B}\,(id, f)$	(56)
Def-map-ana	$\mathsf{T}\, f \quad = \quad [\![\mathsf{B}(f, id) \cdot \mathsf{out}]\!]$	(57)
Absorção-ana	$\mathsf{T}\, f \cdot [\![g]\!] \quad = \quad [\![\mathsf{B}(f, id) \cdot g]\!]$	(58)

MÓNADAS

Multiplicação	$\mu \cdot \mu = \mu \cdot \mathsf{T}\, \mu$	(59)
Unidade	$\mu \cdot u = \mu \cdot \mathsf{T}\, u = id$	(60)
Natural-u	$u \cdot f \quad = \quad \mathsf{T}\, f \cdot u$	(61)
Natural-μ	$\mu \cdot \mathsf{T}\,(\mathsf{T}\, f) \quad = \quad \mathsf{T}\, f \cdot \mu$	(62)
Composição monádica	$f \bullet g \quad = \quad \mu \cdot \mathsf{T}\, f \cdot g$	(63)
Associatividade-$\bullet$	$f \bullet (g \bullet h) = (f \bullet g) \bullet h$	(64)
Identidade-$\bullet$	$u \bullet f = f = f \bullet u$	(65)
Associatividade-$\bullet/\cdot$	$(f \bullet g) \cdot h \quad = \quad f \bullet (g \cdot h)$	(66)
Associatividade-$\cdot/\bullet$	$(f \cdot g) \bullet h \quad = \quad f \bullet (\mathsf{T}\, g \cdot h)$	(67)
μ versus $\bullet$	$id \bullet id \quad = \quad \mu$	(68)

DEFINIÇÕES *ao ponto* ('POINTWISE')

Igualdade extensional	$f = g \quad \Leftrightarrow \quad \langle \forall\, x \ ::\ f\ x = g\ x \rangle$	(69)
Def-comp	$(f \cdot g)\ x \quad = \quad f\ (g\ x)$	(70)
Def-id	$id\ x \quad = \quad x$	(71)
Def-const	$\underline{k}\ x \quad = \quad k$	(72)
Notação-λ	$f\ a = b \quad \equiv \quad f = \lambda a \rightarrow b$	(73)
Def-split	$\langle f, g \rangle\, x \quad = \quad (f\, x, g\, x)$	(74)
Def-$\times$	$(f \times g)\,(a, b) \quad = \quad (f\, a, g\, b)$	(75)
Def-cond	$(p \rightarrow f, g)\, x \quad = \quad \textbf{if}\ p\ x\ \textbf{then}\ f\ x\ \textbf{else}\ g\ x$	(76)
Def-proj	$\pi_1\,(x, y)\ =\ x \qquad \wedge \qquad \pi_2\,(x, y)\ =\ y$	(77)
Elim-let	$\textbf{let}\ x = a\ \textbf{in}\ b \quad = \quad b\,[x/a]$	(78)
Elim-pair	$t \quad = \quad t[(x, y)/z, x/\pi_1 z, y/\pi_2 z]$	(79)
Def-ap	$ap(f, x) = f\ x$	(80)
Curry	$\overline{f}\ a\ b = f\ (a, b)$	(81)
Uncurry	$\widehat{f}\ (a, b) = f\ a\ b$	(82)
Composição monádica	$(f \bullet g)\ a = \textbf{do}\ \{ b \leftarrow g\ a; f\ b \}$	(83)
'Binding as μ'	$x \gg\!= f \quad = \quad (\mu \cdot \mathsf{T}\, f) x$	(84)
Notação-do	$\textbf{do}\ \{ x \leftarrow a; b \} \quad = \quad a \gg\!= (\lambda x \rightarrow b)$	(85)
'μ as binding'	$\mu\, x \quad = \quad x \gg\!= id$	(86)
Sequenciação	$x \gg y \quad = \quad x \gg\!= \underline{y}$	(87)