Exercises 2 : Interaction and Concurrency

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Exercise I.1

Suppose a labelled transition system is given by the following transition relation:

$\{\langle 1,a,2\rangle, \langle 1,a,3\rangle, \langle 2,a,3\rangle, \langle 2,b,1\rangle, \langle 3,a,3\rangle, \langle 3,b,1\rangle, \langle 4,a,5\rangle, \langle 5,a,5\rangle, \langle 5,b,6\rangle, \langle 6,a,5\rangle, \langle 7,a,8\rangle, \langle 8,a,8\rangle, \langle 8,b,7\rangle\}$

Prove or refute $1 \sim 4 \sim 6 \sim 7$.

Exercise I.2

Given two labelled transition systems $\langle S_A, \mathcal{N}, \downarrow_{\mathcal{A}}, \longrightarrow_{\mathcal{A}}\rangle$ and $\langle S_B, \mathcal{N}, \downarrow_{\mathcal{B}}, \longrightarrow_{\mathcal{B}}\rangle$, two states p and q are *mutually similar* iff

$$p \doteqdot q \;\equiv\; p \lesssim q \wedge q \lesssim p$$

1. Show that $\doteqdot$is an equivalence relation.
2. Compare this equivalence with bisimilarity $\sim$.

Exercise I.3

Show that $\sim$is an equivalence relation.

Exercise I.4

Discuss whether bisimilarity $\sim$

- is closed for union
- is closed for intersection

Exercise I.5

A relation R over the state space of a labelled transition system is a *word bisimulation* if, whenever $\langle p, q\rangle \in R$ and $s \in \mathcal{N}^*$, we have

$$p \xrightarrow{s} p' \;\Rightarrow\; \langle \exists\, q' \;:\; q' \in S_2 :\; q \xrightarrow{s} q' \wedge \langle p', q'\rangle \in R\rangle$$
$$q \xrightarrow{s} q' \;\Rightarrow\; \langle \exists\, p' \;:\; p' \in S_1 :\; p \xrightarrow{s} p' \wedge \langle p', q'\rangle \in R\rangle$$

1. Define formally relation $\xrightarrow{s}$, for $s \in \mathcal{N}^*$
2. Two states are *word bisimilar* iff they belong to a word bisimulation. Show that two states p and q are word bisimilar iff $p \sim q$.