Labelled Transition Systems (II)

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Simulation

Intuition

A state q simulates another state p (in the same or in a different LTS) if every transition from q is corresponded by a transition from p and this capacity is kept along the whole life of the system to which state space q belongs to.

Simulation

Definition

Given LTS $\langle S_1, T_1\rangle$ and $\langle S_2, T_2\rangle$ over $\mathcal{N}$, relation $R \subseteq S_1 \times S_2$ is a simulation iff, whenever $\langle p, q\rangle \in R$ and $a \in \mathcal{N}$,

$$p \xrightarrow{a} p' \;\Rightarrow\; \langle \exists\, q' \;:\; q' \in S_2 :\; q \xrightarrow{a} q' \wedge \langle p', q'\rangle \in R\rangle$$

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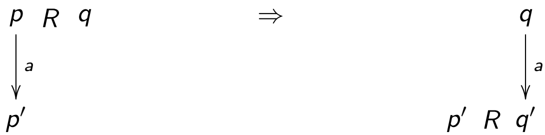
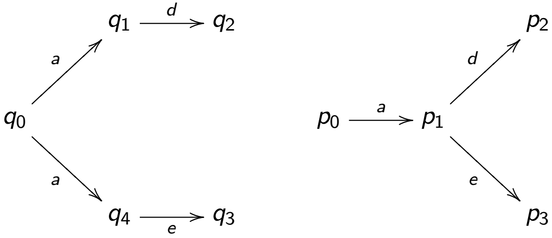


Example



$$q_0 \lesssim p_0 \qquad \text{cf.} \qquad \{\langle q_0, p_0\rangle, \langle q_1, p_1\rangle, \langle q_4, p_1\rangle, \langle q_2, p_2\rangle, \langle q_3, p_3\rangle\}$$

Similarity

Definition

$$p \lesssim q \;\Leftrightarrow\; \langle \exists\, R \;::\; R \text{ is a simulation and } \langle p, q \rangle \in R \rangle$$

Lemma

The similarity relation is a preorder
(ie, reflexive and transitive)

Bisimulation

Definition

Given LTS $\langle S_1, T_1\rangle$ and $\langle S_2, T_2\rangle$ over $\mathcal{N}$, relation $R \subseteq S_1 \times S_2$ is a bisimulation iff both R and its converse R° are simulations.
I.e., whenever $\langle p, q\rangle \in R$ and $a \in \mathcal{N}$,

$$p \xrightarrow{a} p' \;\Rightarrow\; \langle \exists\, q' \;:\; q' \in S_2 :\; q \xrightarrow{a} q' \wedge \langle p', q'\rangle \in R\rangle$$
$$q \xrightarrow{a} q' \;\Rightarrow\; \langle \exists\, p' \;:\; p' \in S_1 :\; p \xrightarrow{a} p' \wedge \langle p', q'\rangle \in R\rangle$$

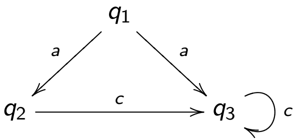
Bisimulation

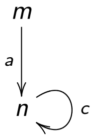
The Game characterization

Two players R and I discuss whether the transition structures are mutually corresponding

- R starts by chosing a transition
- I replies trying to match it
- if I succeeds, R plays again
- R wins if I fails to find a corresponding match
- I wins if it replies to all moves from R and the game is in a configuration where all states have been visited or R can't move further. In this case is said that I has a wining strategy

Examples





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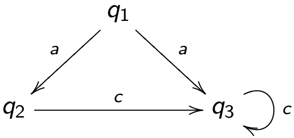

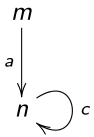

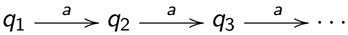

Bisimilarity

Definition

$$p \sim q \;\Leftrightarrow\; \langle \exists\, R \;::\; R \text{ is a bisimulation and } \langle p, q \rangle \in R \rangle$$

Lemma

1. The identity relation id is a bisimulation
2. The empty relation $\perp$ is a bisimulation
3. The converse R° of a bisimulation is a bisimulation
4. The composition $S \cdot R$ of two bisimulations S and R is a bisimulation
5. The $\bigcup_{i \in I} R_i$ of a family of bisimulations $\{R_i \mid i \in I\}$ is a bisimulation

Bisimilarity

Lemma

The bisimilarity relation is an equivalence relation
(ie, reflexive, symmetric and transitive)

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The class of all bisimulations between two LTS has the structure of a complete lattice, ordered by set inclusion, whose top is the bisimilarity relation $\sim$.

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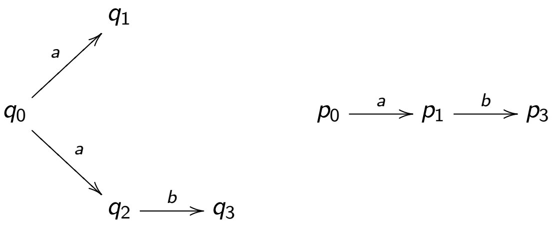
Bisimilarity

Warning

The bisimilarity relation $\sim$ is not the symmetric closure of $\lesssim$

Example

$$q_0 \lesssim p_0, \; p_0 \lesssim q_0 \quad \text{but} \quad p_0 \not\sim q_0$$



After thoughts

Similarity as the greatest simulation

$$\lesssim \triangleq \bigcup\{S \mid S \text{ is a simulation}\}$$

Bisimilarity as the greatest bisimulation

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cf relational translation of definitions
$\lesssim$ and $\sim$ as greatest fix points (Tarski's theorem)

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The questions to follow ...

- We already have a semantic model for reactive systems. With which language shall we describe them?
- How to compare and transform such systems?
- How to express and prove their proprieties?

$\rightsquigarrow$ process languages and calculi
cf. CCS (Milner, 80), CSP (Hoare, 85),
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